

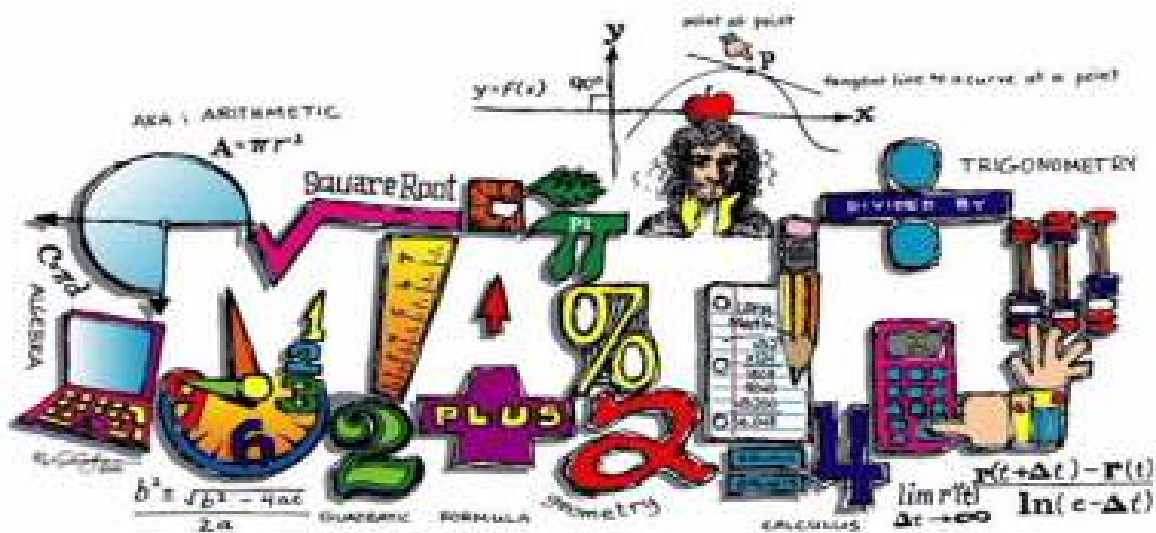
Name _____

Period _____

Algebra 1 Honors Incoming Assignment

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Bak MSOA Summer Required Mathematics Assignment Directions:



NO CALCULATOR!

Complete the ODD Problems, only, on each page.

Show all appropriate work and circle your answers.

The packet collection will not occur during the first day of school.

This will be a part of your first nine weeks Assignment grade.

WE LOOK FORWARD TO SEEING YOU IN AUGUST. 😊

Reteach

Divide Rational Numbers

Two numbers with a product of 1 are **multiplicative inverses**, or **reciprocals**, of each other.

Example 1 Write the multiplicative inverse of $-2\frac{3}{4}$.

$-2\frac{3}{4} = -\frac{11}{4}$ Write $-2\frac{3}{4}$ as an improper fraction.
 Since $-\frac{11}{4} \left(-\frac{4}{11}\right) = 1$, the multiplicative inverse of $-2\frac{3}{4}$ is $-\frac{4}{11}$.

To divide by a fraction, multiply by its multiplicative inverse.

Example 2 Find $\frac{3}{8} \div \frac{6}{7}$. Write in simplest form.

$\frac{3}{8} \div \frac{6}{7} = \frac{3}{8} \cdot \frac{7}{6}$ Multiply by the multiplicative inverse of $\frac{6}{7}$, which is $\frac{7}{6}$.
 $= \frac{\cancel{3}^1}{8} \cdot \frac{7}{\cancel{6}_2}$ Divide 6 and 3 by their GCF, 3.
 $= \frac{7}{16}$ Multiply.

Exercises

Write the multiplicative inverse of each number.

1. $\frac{3}{5}$

2. $-\frac{8}{9}$

3. $\frac{1}{10}$

4. $-\frac{1}{6}$

5. $2\frac{3}{5}$

6. $-1\frac{2}{3}$

7. $-5\frac{2}{5}$

8. $7\frac{1}{4}$

Divide. Write in simplest form.

9. $\frac{1}{3} \div \frac{1}{6}$

10. $\frac{2}{5} \div \frac{4}{7}$

11. $-\frac{5}{6} \div \frac{3}{4}$

12. $1\frac{1}{5} \div 2\frac{1}{4}$

13. $3\frac{1}{7} \div \left(-3\frac{2}{3}\right)$

14. $-\frac{4}{9} \div 2$

15. $\frac{6}{11} \div (-4)$

16. $5 \div 2\frac{1}{3}$

Reteach

Variables and Expressions

To evaluate an algebraic expression you replace each variable with its numerical value, then use the order of operations to simplify.

Example 1 Evaluate $5m - 3n$ if $m = 6$ and $n = -5$.

$$\begin{aligned} 5m - 3n &= 5(6) - 3(-5) \\ &= 30 - (-15) \\ &= 45 \end{aligned}$$

Replace m with 6 and n with -5 .
Use the order of operations.
Subtract -15 from 30.

Example 2 Evaluate $\left(\frac{3 + ab}{3}\right)$ if $a = 7$ and $b = 6$.

$$\begin{aligned} \frac{3 + ab}{3} &= \frac{3 + (7)(6)}{3} \\ &= \frac{45}{3} \\ &= 15 \end{aligned}$$

Replace a with 7 and b with 6.
The fraction bar is like a grouping symbol.
Divide.

Example 3 Evaluate $6(x + y) - 4$ if $x = 8$ and $y = 3$.

$$\begin{aligned} 6(x + y) - 4 &= 6(8 + 3) - 4 \\ &= 6(11) - 4 \\ &= 62 \end{aligned}$$

Replace x with 8 and y with 3.
Use the order of operations.
Subtract 4 from 66.

Example 4 Translate each phrase into an algebraic expression.

a. twelve dollars less than Tamika has

Let d represent the money Tamika has. The expression is $d - 12$.

b. three less than twice the number of students

Let n represent the number of students. The expression is $2n - 3$.

Exercises

Evaluate each expression if $a = 4$, $b = 2$, and $c = -3$.

1. $3ac$

2. $5b$

3. abc

4. $5a + 6c$

5. $\frac{ab}{8}$

6. $2a - 3b$

7. $\frac{ac}{b}$

8. $6a - b$

9. $20 - \frac{ac}{b} + 2$

10. $2bc$

11. $\frac{ac - 3b}{b}$

12. $\frac{6 + 3b}{2a - 2}$

Translate each phrase into an algebraic expression.

13. four more than the number of DVDs Joan has

14. six less than five times the number of miles

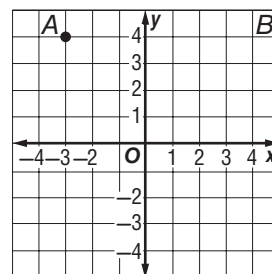
Reteach

Ordered Pairs and Relations

Example 1 Name the ordered pair for point A.

- Start at the origin.
- Move left on the x -axis to find the x -coordinate of point A, which is -3 .
- Move up the y -axis to find the y -coordinate, which is 4 .

So, the ordered pair for point A is $(-3, 4)$.



Example 2 Graph point B at $(5, 4)$.

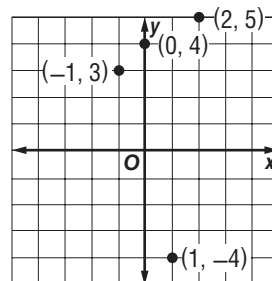
- Use the coordinate plane shown above. Start at the origin and move 5 units to the right. Then move up 4 units.
- Draw a dot and label it $B(5, 4)$.

Example 3 Express the relation $\{(2, 5), (-1, 3), (0, 4), (1, -4)\}$ as a table and a graph. Then state the domain and range.

The domain is $\{-1, 0, 1, 2\}$.

The range is $\{-4, 3, 4, 5\}$.

x	y
2	5
-1	3
0	4
1	-4



Exercises

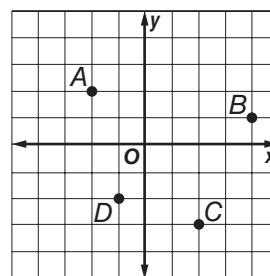
Name the ordered pair for each point.

1. A

2. B

3. C

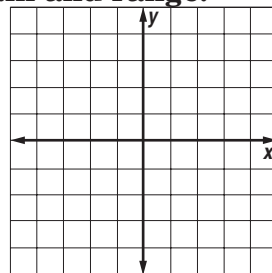
4. D



Express the relation as a table and a graph. Then state the domain and range.

5. $\{(-3, 1), (2, 4), (-1, 0), (4, -4)\}$

x	y



Reteach

Functions

A **function** is a relation in which each member of the domain (input value) is paired with exactly one member of the range (output value). You can organize the input, rule, and output of a function using a function table.

Example 1 Choose four values for x to make a function table for $f(x) = 2x + 4$. Then state the domain and range of the function.

Substitute each domain value x , into the function rule. Then simplify to find the range value.

$$f(x) = 2x + 4$$

$$f(-1) = 2(-1) + 4 \text{ or } 2$$

$$f(0) = 2(0) + 4 \text{ or } 4$$

$$f(1) = 2(1) + 4 \text{ or } 6$$

$$f(2) = 2(2) + 4 \text{ or } 8$$

Input x	Rule $2x + 4$	Output $f(x)$
-1	$2(-1) + 4$	2
0	$2(0) + 4$	4
1	$2(1) + 4$	6
2	$2(2) + 4$	8

The domain is $\{-1, 0, 1, 2\}$. The range is $\{2, 4, 6, 8\}$.

Exercises

Find each function value.

1. $f(1)$ if $f(x) = x + 3$

2. $f(6)$ if $f(x) = 2x$

3. $f(4)$ if $f(x) = 5x - 4$

4. $f(9)$ if $f(x) = -3x + 10$

5. $f(-2)$ if $f(x) = 4x - 1$

6. $f(-5)$ if $f(x) = -2x + 8$

Choose four values for x to make a function table for each function. Then state the domain and range of the function.

7. $f(x) = x - 10$

8. $f(x) = 2x + 6$

9. $f(x) = 2 - 3x$

x	$x - 10$	$f(x)$

x	$2x + 6$	$f(x)$

x	$2 - 3x$	$f(x)$

Reteach

Linear Functions

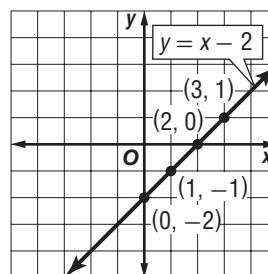
A function in which the graph of the solutions forms a line is called a **linear function**. A linear function can be represented by an equation, a table, a set of ordered pairs, or a graph.

Example 1 Graph $y = x - 2$.

Step 1 Choose some values for x .
Use these values to make a function table.

x	$x - 2$	y	(x, y)
0	$0 - 2$	-2	$(0, -2)$
1	$1 - 2$	-1	$(1, -1)$
2	$2 - 2$	0	$(2, 0)$
3	$3 - 2$	1	$(3, 1)$

Step 2 Graph each ordered pair on a coordinate plane.
Draw a line that passes through the points.
The line is the graph of the linear function.

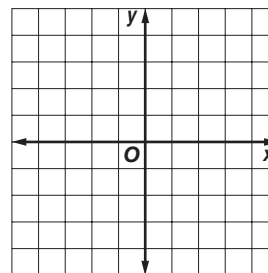


Exercises

Complete the function table. Then graph the function.

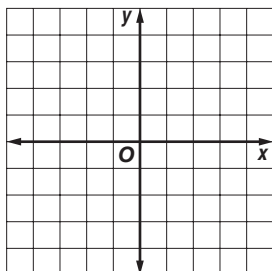
1. $y = x + 3$

x	$x + 3$	y	(x, y)
-2			
0			
1			
2			

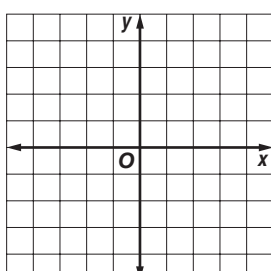


Graph each function.

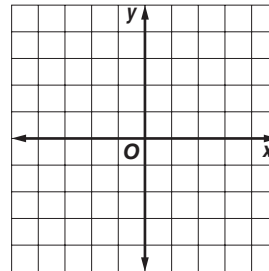
2. $y = 3x + 2$



3. $y = 2 - x$



4. $y = 3x - 1$



Determine whether each set of data is continuous or discrete.

5. the size of airmail packages

6. the number of boxes in an airmail shipment

Reteach

Linear and Nonlinear Functions

Linear functions represent constant rates of change. The rate of change for nonlinear functions is not constant. That is, the values do not increase or decrease at the same rate. You can use a table to determine if the rate of change is constant.

Example 1 Determine whether the table represents a *linear* or a *nonlinear* function. Explain.

		+2	+2	+2	
		↘	↘	↘	
<i>x</i>	3	5	7	9	
<i>y</i>	7	10	13	16	
		↗	↗	↗	
		+3	+3	+3	

As x increases by 2, y increases by 3. The rate of change is constant, so this function is linear.

Example 2 Determine whether the table represents a *linear* or a *nonlinear* function. Explain.

		+1	+1	+1	
		↘	↘	↘	
<i>x</i>	1	2	3	4	
<i>y</i>	-3	-6	-10	-15	
		↗	↗	↗	
		-3	-4	-5	

As x increases by 1, y decreases by a different amount each time. The rate of change is not constant, so this function is nonlinear.

Exercises

Determine whether each table represents a *linear* or a *nonlinear* function. Explain.

1.

<i>x</i>	3	5	7	9
<i>y</i>	7	9	11	13

2.

<i>x</i>	1	5	9	13
<i>y</i>	0	6	8	9

3.

<i>x</i>	3	6	9	12
<i>y</i>	2	3	4	5

4.

<i>x</i>	-2	-3	-4	-5
<i>y</i>	-1	-5	9	8

Reteach

Constant Rate of Change

Relationships that have straight-lined graphs are called **linear relationships**. The rate of change between any two points in a linear relationship is the same, or constant. A linear relationship has a **constant rate of change**.

Example

The height of a hot air balloon after a few seconds is shown. Determine whether the relationship between the two quantities is linear. If so, find the constant rate of change. If not, explain your reasoning.

Time (sec)	Height of Hot Air Balloon (ft)
1	9
2	18
3	27
4	36

As the number of seconds increase by 1, the height of the balloon increases by 9 feet.

Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{9}{1}$ or 9 feet per second. This means that the balloon is rising 9 feet per second.

Exercises

Determine whether the relationship between the two quantities described in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

1.

Greeting Cards	
Number of Cards	Total Cost(\$)
1	1.50
2	3.00
3	4.50
4	6.00

2.

Party Table Rental	
Number of Tables	Cost(\$)
1	10
2	18
3	24
4	28

3.

Donuts	
Dozens Bought	Cost (\$)
2	3.25
4	6.50
6	9.75
8	13.00

4.

Running	
Time (min)	Distance(mi)
15	2
30	4
45	5
60	6

Reteach

Slope

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y -coordinates to the corresponding difference in the x -coordinates. As an equation, the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

Example 1 Find the slope of the line that passes through $A(-1, -1)$ and $B(2, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope formula

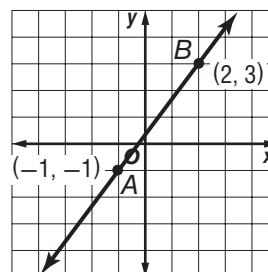
$$m = \frac{3 - (-1)}{2 - (-1)}$$

$$(x_1, y_1) = (-1, -1),$$

$$(x_2, y_2) = (2, 3)$$

$$m = \frac{4}{3}$$

Simplify.



Check When going from left to right, the graph of the line slants upward. This is correct for a positive slope.

Example 2 Find the slope of the line that passes through $C(1, 4)$ and $D(3, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope formula

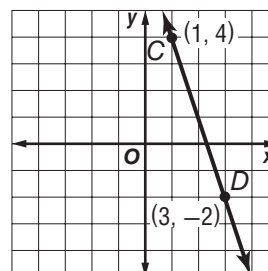
$$m = \frac{-2 - 4}{3 - 1}$$

$$(x_1, y_1) = (1, 4),$$

$$(x_2, y_2) = (3, -2)$$

$$m = \frac{-6}{2} \text{ or } -3$$

Simplify.



Check When going from left to right, the graph of the line slants downward. This is correct for a negative slope.

Exercises

Find the slope of the line that passes through each pair of points.

1. $A(0, 1), B(3, 4)$

2. $C(1, -2), D(3, 2)$

3. $E(4, -4), F(2, 2)$

4. $G(3, 1), H(6, 3)$

5. $I(4, 3), J(2, 4)$

6. $K(-4, 4), L(5, 4)$

Reteach

Slope-Intercept Form

Linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y -intercept.

Example 1 State the slope and the y -intercept of the graph of $y = x - 3$.

$$\begin{array}{rcl}
 y & = & x - 3 \\
 y & = & 1x + (-3) \\
 \uparrow & & \uparrow \\
 y & = & mx + b
 \end{array}
 \quad
 \begin{array}{l}
 \text{Write the original equation.} \\
 \text{Write the equation in the form } y = mx + b. \\
 m = 1, b = -3
 \end{array}$$

The slope of the graph is 1, and the y -intercept is -3 .

You can use the slope-intercept form of an equation to graph the equation.

Example 2 Graph $y = 2x + 1$ using the slope and y -intercept.

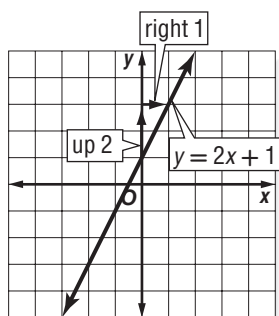
Step 1 Find the slope and y -intercept.

$$y = 2x + 1 \quad \text{slope} = 2, \text{ } y\text{-intercept} = 1$$

Step 2 Graph the y -intercept 1.

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$$m = \frac{2}{1} \leftarrow \begin{array}{l} \text{change in } y : \text{up 2 units} \\ \text{change in } x : \text{right 1 unit} \end{array}$$



Step 4 Draw a line through the two points.

Exercises

State the slope and the y -intercept for the graph of each equation.

1. $y = x + 1$

2. $y = 2x - 4$

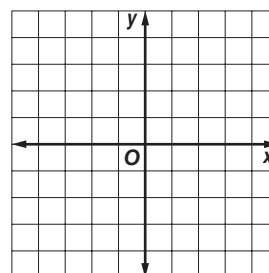
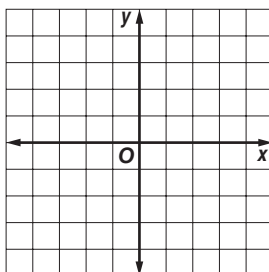
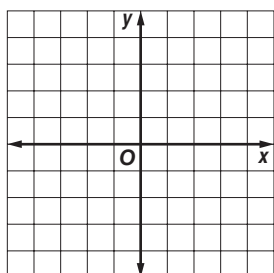
3. $y = \frac{1}{2}x - 1$

Graph each equation using the slope and the y -intercept.

4. $y = 2x + 2$

5. $y = x - 1$

6. $y = \frac{1}{2}x + 2$

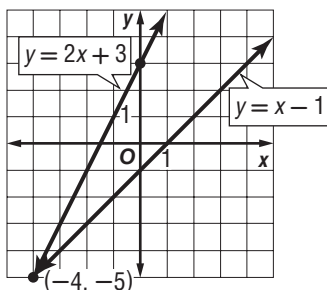


Reteach

Solve Systems of Equations by Graphing

Example Solve the system $y = 2x + 3$ and $y = x - 1$ by graphing.

Graph each equation on the same coordinate plane.



The graphs appear to intersect at $(-4, -5)$.

Check this estimate by replacing x with -4 and y with -5 .

Check

$$y = 2x + 3$$

$$-5 \stackrel{?}{=} 2(-4) + 3$$

$$-5 = -5 \checkmark$$

$$y = x - 1$$

$$-5 \stackrel{?}{=} -4 - 1$$

$$-5 = -5 \checkmark$$

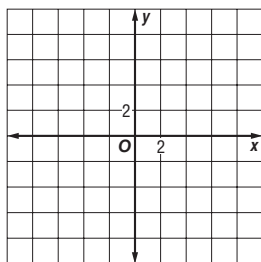
The solution of the system is $(-4, -5)$.

Exercises

Solve each system of equations by graphing.

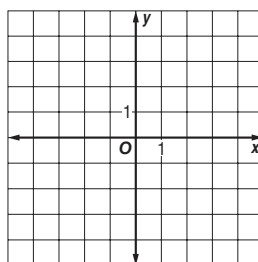
1. $y = 2x + 5$

$y = -x + 8$



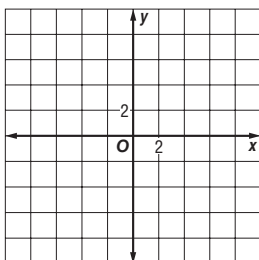
2. $y = -x - 3$

$y = x + 1$



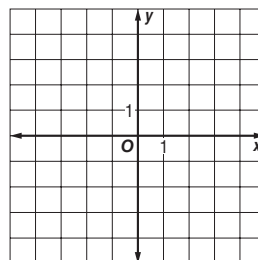
3. $y = -3x + 9$

$y = -3x + 3$



4. $y = -2x + 4$

$y = -x + 3$



Reteach

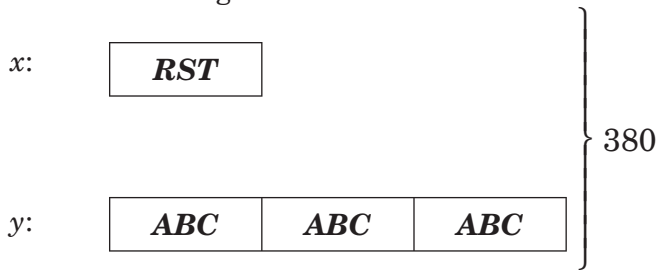
Solve Systems of Equations by Substitution

Real-World Example

You own three times as many shares of *ABC* stock as you do of *RST* stock. Altogether you have 380 shares of stock.

- a. Write a system of equations to represent this situation.

Draw a bar diagram.



Use the diagram to write the system.

$y = 3x$ There are 3 times as many shares *ABC* stocks as *RST* stocks.

$x + y = 380$ The total number of stocks owned is 380.

- b. Solve the system by substitution. Interpret the solution.

Since y is equal to $3x$, you can replace y with $3x$ in the second equation.

$x + y = 380$ Write the equation.

$x + 3x = 380$ Replace y with $3x$.

$4x = 380$ Simplify.

$\frac{4x}{4} = \frac{380}{4}$ Division Property of Equality

$x = 95$ Simplify.

Since $x = 95$ and $y = 3x$, then $y = 285$ when $x = 95$. The solution of this system of equations is $(95, 285)$. This means that you own 95 shares of *RST* stock and 285 shares of *ABC* stock.

Exercises

Solve each system of equations by substitution.

1. $y = x + 3$
 $y = 4x$

2. $y = -x - 2$
 $y = -2x$

3. $y = x + 14$
 $y = 8x$

4. $y = x - 6$
 $y = 2x$

5. $y = -x + 8$
 $y = 3x$

6. $y = -x$
 $y = -2x$

Reteach

Write Equations

The table shows several verbal phrases for each algebraic expression.

Phrases	Expression	Phrases	Expression
8 more than a number the sum of 8 and a number x plus 8 x increased by 8	$x + 8$	the difference of r and 6 6 subtracted from a number 6 less than a number r minus 6	$r - 6$
Phrases	Expression	Phrases	Expression
4 multiplied by n 4 times a number the product of 4 and n	$4n$	a number divided by 3 the quotient of z and 3 the ratio of z and 3	$\frac{z}{3}$

The table shows several verbal sentences that represent the same equation.

Sentences	Equation
9 less than a number is equal to 45. The difference of a number and 9 is 45. A number decreased by 9 is 45. 45 is equal to a number minus 9.	$n - 9 = 45$

Write each verbal phrase as an algebraic expression.

- the sum of 8 and t
- the quotient of g and 15
- the product of 5 and b
- p increased by 10
- 14 less than f
- the difference of 32 and x

Define a variable. Then write an equation to model each situation.

- 5 more than a number is 6.
- The product of 7 and b is equal to 63.
- The sum of r and 45 is 79.
- The quotient of x and 7 is equal to 13.
- The original price decreased by \$5 is \$34.
- 5 shirts at \$ d each is \$105.65.

Reteach

Solve Addition and Subtraction Equations

You can use the following properties to solve addition and subtraction equations.

- *Addition Property of Equality* — If you add the same number to each side of an equation, the two sides remain equal.
- *Subtraction Property of Equality* — If you subtract the same number from each side of an equation, the two sides remain equal.

Example 1 Solve $w + 19 = 45$. Check your solution.

$$w + 19 = 45$$

Write the equation.

$$\begin{array}{r} w + 19 = 45 \\ - 19 = -19 \\ \hline \end{array}$$

Subtraction Property of Equality

$$w = 26$$

Simplify.

Check $w + 19 = 45$

Write the original equation.

$$26 + 19 \stackrel{?}{=} 45$$

Replace w with 26. Is this sentence true?

$$45 = 45 \checkmark$$

The sentence is true.

Example 2 Solve $h - 25 = -76$. Check your solution.

$$h - 25 = -76$$

Write the equation.

$$\begin{array}{r} h - 25 = -76 \\ + 25 = +25 \\ \hline \end{array}$$

Addition Property of Equality

$$h = -51$$

Simplify.

Check $h - 25 = -76$

Write the original equation.

$$-51 - 25 \stackrel{?}{=} -76$$

Replace h with -51. Is this sentence true?

$$-76 = -76 \checkmark$$

The sentence is true.

Exercises

Solve each equation. Check your solution.

1. $s - 4 = 12$

2. $d + 2 = 21$

3. $h + 6 = 15$

4. $x + 5 = -8$

5. $b - 10 = -34$

6. $f - 22 = -6$

7. $17 + c = 41$

8. $v - 36 = 25$

9. $y - 29 = -51$

10. $19 = z - 32$

11. $13 + t = -29$

12. $55 = 39 + k$

13. $62 + b = 45$

14. $x - 39 = -65$

15. $-56 = -47 + n$

Reteach**Solve Multiplication and Division Equations**

You can use the following properties to solve multiplication and division equations.

- **Multiplication Property of Equality** — If you multiply each side of an equation by the same number, the two sides remain equal.
- **Division Property of Equality** — If you divide each side of an equation by the same nonzero number, the two sides remain equal.

Example 1 Solve $19w = 114$. Check your solution.

$$19w = 114$$

Write the equation.

$$\frac{19w}{19} = \frac{114}{19}$$

Division Property of Equality

$$w = 6$$

Simplify.

Check $19w = 114$

Write the original equation.

$$19(6) \stackrel{?}{=} 114$$

Replace w with 6.

$$114 = 114 \checkmark$$

This sentence is true.

Example 2 Solve $\frac{d}{15} = -9$. Check your solution.

$$\frac{d}{15} = -9$$

Write the equation.

$$\frac{d}{15} (15) = -9(15)$$

Multiplication Property of Equality

$$d = -135$$

Simplify.

Check $\frac{d}{15} = -9$

Write the original equation.

$$\frac{-135}{15} \stackrel{?}{=} -9$$

Replace d with -135 .

$$-9 = -9 \checkmark$$

This sentence is true.

Exercises

Solve each equation. Check your solution.

1. $\frac{r}{5} = 6$

2. $2d = 12$

3. $7h = -21$

4. $-8x = 40$

5. $\frac{f}{8} = -6$

6. $\frac{x}{-10} = -7$

7. $17c = -68$

8. $\frac{h}{-11} = 12$

9. $29t = -145$

10. $125 = 5z$

11. $13t = -182$

12. $117 = -39k$

Reteach

Solve Two-Step Equations

A **two-step equation** contains two operations. To solve a two-step equation, undo each operation in reverse order.

Example 1 Solve $-2a + 6 = 14$. Check your solution.

$$-2a + 6 = 14$$

Write the equation.

$$-6 = -6$$

Subtraction Property of Equality

$$-2a = 8$$

Simplify.

$$\frac{-2a}{-2} = \frac{8}{-2}$$

Division Property of Equality

$$a = -4$$

Simplify.

Check

$$-2a + 6 = 14$$

Write the equation.

$$-2(-4) + 6 \stackrel{?}{=} 14$$

Replace a with -4 to see if the sentence is true.

$$14 = 14 \checkmark$$

The sentence is true.

The solution is -4 .

Sometimes it is necessary to combine like terms before solving an equation.

Example 2 Solve $5 = 8x - 2x - 7$. Check your solution.

$$5 = 8x - 2x - 7$$

Write the equation.

$$5 = 6x - 7$$

Combine like terms.

$$5 + 7 = 6x - 7 + 7$$

Addition Property of Equality

$$12 = 6x$$

Simplify.

$$\frac{12}{6} = \frac{6x}{6}$$

Division Property of Equality

$$2 = x$$

Simplify.

The solution is 2 .

Check this solution.

Exercises

Solve each equation. Check your solution.

1. $2d + 7 = 9$

2. $11 = 3z + 5$

3. $2s - 4 = 6$

4. $-12 = 5r + 8$

5. $-6p - 3 = 9$

6. $-14 = 3x + x - 2$

7. $5c + 2 - 3c = 10$

8. $3 + 7n + 2n = 21$

9. $21 = 6r + 5 - 7r$

10. $8 - 5b = -7$

11. $-10 = 6 - 4m$

12. $-3t + 4 = 19$

13. $2 + \frac{a}{6} = 5$

14. $-\frac{1}{3}q - 7 = -3$

15. $4 - \frac{v}{5} = 0$

Reteach

Write Two-Step Equations

Some verbal sentences translate into two-step equations.

Example 1 Translate each sentence into an equation.

Sentence

Equation

Four more than three times a number is 19.

$$3n + 4 = 19$$

Five is seven less than twice a number.

$$5 = 2n - 7$$

Seven more than the quotient of a number and 3 is 10.

$$7 + \frac{n}{3} = 10$$

After a sentence has been translated into a two-step equation, you can solve the equation.

Example 2 Translate the sentence into an equation. Then find the number. Thirteen more than five times a number is 28.

Words Thirteen more than five times a number is 28.

Variable Let n = the number.

Equation $5n + 13 = 28$

Write the equation.

$$\begin{array}{r} 5n + 13 = 28 \\ -13 = -13 \\ \hline 5n = 15 \end{array}$$

Subtraction Property of Equality

$$5n = 15$$

Simplify.

$$\frac{5n}{5} = \frac{15}{5}$$

Division Property of Equality

$$n = 3$$

Therefore, the number is 3.

Exercises

Translate each sentence into an equation. Then find each number.

- Five more than twice a number is 7.
- Fourteen more than three times a number is 2.
- Seven less than twice a number is 5.
- Two more than four times a number is -10 .
- Eight less than three times a number is -14 .
- Three more than the quotient of a number and 2 is 7.

Reteach

Powers and Exponents

The product of repeated factors can be expressed as a **power**. A power consists of a **base** and an **exponent**. The exponent tells how many times the base is used as a factor.

Example 1 Write each expression using exponents.

a. $7 \cdot 7 \cdot 7 \cdot 7$

$$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$$

The number 7 is a factor 4 times. So, 7 is the base and 4 is the exponent.

b. $y \cdot y \cdot x \cdot y \cdot x$

$$y \cdot y \cdot x \cdot y \cdot x = y \cdot y \cdot y \cdot x \cdot x$$

Commutative Property

$$= (y \cdot y \cdot y) \cdot (x \cdot x)$$

Associative Property

$$= y^3 \cdot x^2$$

Definition of exponents

To evaluate a power, perform the repeated multiplication to find the product.

Example 2 Evaluate $(-6)^4$.

$$(-6)^4 = (-6) \cdot (-6) \cdot (-6) \cdot (-6)$$

Write the power as a product.

$$= 1,296$$

Multiply.

The order of operations states that exponents are evaluated before multiplication, division, addition, and subtraction.

Example 3 Evaluate $m^2 + (n - m)^3$ if $m = -3$ and $n = 2$.

$$m^2 + (n - m)^3 = (-3)^2 + (2 - (-3))^3$$

Replace m with -3 and n with 2 .

$$= (-3)^2 + (5)^3$$

Perform operations inside parentheses.

$$= (-3 \cdot -3) + (5 \cdot 5 \cdot 5)$$

Write the powers as products.

$$= 9 + 125 \text{ or } 134$$

Add.

Exercises

Write each expression using exponents.

1. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

2. $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

3. $5 \cdot 5 \cdot 9 \cdot 9 \cdot 5 \cdot 9 \cdot 5 \cdot 5$

Evaluate each expression.

4. 2^4

5. $(-3)^5$

6. $\left(\frac{3}{4}\right)^3$

ALGEBRA Evaluate each expression if $a = 5$ and $b = -4$.

7. $a^2 + b^2$

8. $(a + b)^2$

9. $a + b^2$

Reteach

Multiply and Divide Monomials

The **Product of Powers** rule states that to multiply powers with the same base, add their exponents.

Example 1 Simplify. Express using exponents.

a. $2^3 \cdot 2^2$

$$\begin{aligned} 2^3 \cdot 2^2 &= 2^{3+2} \\ &= 2^5 \end{aligned}$$

The common base is 2.

Add the exponents.

b. $2s^6(7s^7)$

$$\begin{aligned} 2s^6(7s^7) &= (2 \cdot 7)(s^6 \cdot s^7) \\ &= 14(s^{6+7}) \\ &= 14s^{13} \end{aligned}$$

Commutative and Associative Properties

The common base is s.

Add the exponents.

The **Quotient of Powers** rule states that to divide powers with the same base, subtract their exponents.

Example 2 Simplify $\frac{k^8}{k^1}$. Express using exponents.

$$\begin{aligned} \frac{k^8}{k^1} &= k^{8-1} \\ &= k^7 \end{aligned}$$

The common base is k.

Subtract the exponents.

Example 3 Simplify $\frac{(-2)^{10} \cdot 5^6 \cdot 6^3}{(-2)^6 \cdot 5^3 \cdot 6^2}$.

$$\begin{aligned} \frac{(-2)^{10} \cdot 5^6 \cdot 6^3}{(-2)^6 \cdot 5^3 \cdot 6^2} &= \left(\frac{(-2)^{10}}{(-2)^6} \right) \cdot \left(\frac{5^6}{5^3} \right) \cdot \left(\frac{6^3}{6^2} \right) \\ &= (-2)^4 \cdot 5^3 \cdot 6^1 \\ &= 16 \cdot 125 \cdot 6 \text{ or } 12,000 \end{aligned}$$

Group by common base.

Subtract the exponents.

Simplify.

Exercises

Simplify. Express using exponents.

1. $5^2 \cdot 5^5$

2. $e^2 \cdot e^7$

3. $2a^5 \cdot 6a$

4. $4x^2(-5x^6)$

5. $\frac{7^9}{7^3}$

6. $\frac{v^{14}}{v^6}$

7. $\frac{15w^7}{5w^2}$

8. $\frac{10m^8}{2m}$

9. $\frac{2^5 \cdot 3^7 \cdot 4^3}{2^1 \cdot 3^5 \cdot 4}$

10. $\frac{4^{15} \cdot (-5)^6}{4^{12} \cdot (-5)^4}$

Reteach***Powers of Monomials*****Power of a Power:** To find the power of a power, multiply the exponents.**Power of a Product:** To find the power of a product, find the power of each factor and multiply.**Example 1** Simplify $(5^3)^6$.

$$\begin{aligned}(5^3)^6 &= 5^{3 \cdot 6} && \text{Power of a power} \\ &= 5^{18} && \text{Simplify.}\end{aligned}$$

Example 2 Simplify $(-3m^2n^4)^3$.

$$\begin{aligned}(-3m^2n^4)^3 &= (-3)^3 \cdot m^{2 \cdot 3} \cdot n^{4 \cdot 3} && \text{Power of a product} \\ &= -27m^6n^{12} && \text{Simplify.}\end{aligned}$$

Exercises**Simplify.**

1. $(4^3)^5$

2. $(4^2)^7$

3. $(9^2)^4$

4. $(k^4)^2$

5. $[(6^3)^2]^2$

6. $[(3^2)^2]^3$

7. $(5q^4r^2)^5$

8. $(3y^2z^2)^6$

9. $(7a^4b^3c^7)^2$

10. $(-4d^3e^5)^2$

11. $(-5g^4h^9)^7$

12. $(0.2k^8)^2$

Reteach

Negative Exponents

Any nonzero number to the zero power is 1. Any nonzero number to the negative n power is the multiplicative inverse of the number to the n th power.

Example 1 Write each expression using a positive exponent.

a. 7^{-3}

$$7^{-3} = \frac{1}{7^3} \quad \text{Definition of negative exponent}$$

b. a^{-4}

$$a^{-4} = \frac{1}{a^4} \quad \text{Definition of negative exponent}$$

Example 2 Evaluate each expression.

a. 5^{-4}

$$\begin{aligned} 5^{-4} &= \frac{1}{5^4} && \text{Definition of negative exponent} \\ &= \frac{1}{625} && 5^4 = 5 \cdot 5 \cdot 5 \cdot 5 \end{aligned}$$

b. $(-3)^{-5}$

$$\begin{aligned} (-3)^{-5} &= \frac{1}{(-3)^5} && \text{Definition of negative exponent} \\ &= \frac{1}{-243} && (-3)^5 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) \end{aligned}$$

Example 3

Write $\frac{1}{6^5}$ as an expression using a negative exponent.

$$\frac{1}{6^5} = 6^{-5} \quad \text{Definition of negative exponent}$$

Example 4 Simplify. Express using positive exponents.

a. $x^{-3} \cdot x^5$

$$\begin{aligned} x^{-3} \cdot x^5 &= x^{(-3) + 5} && \text{Product of Powers} \\ &= x^2 && \text{Add the exponents.} \end{aligned}$$

b. $\frac{w^{-5}}{w^{-7}}$

$$\begin{aligned} \frac{w^{-5}}{w^{-7}} &= w^{-5 - (-7)} && \text{Quotient of Powers} \\ &= w^2 && \text{Subtract the exponents.} \end{aligned}$$

Exercises

Write each expression using a positive exponent.

1. a^{-8}

2. 6^{-3}

3. n^{-4}

Evaluate each expression.

4. 7^{-2}

5. 9^{-3}

6. $(-2)^{-5}$

Write each fraction as an expression using a negative exponent.

7. $\frac{1}{5^7}$

8. $\frac{1}{3^6}$

9. $\frac{1}{x^8}$

Simplify. Express using positive exponents.

10. $4^{-2} \cdot 4^{-4}$

11. $r^{-3} \cdot r^5$

12. $\frac{h^{-2}}{h^4}$

Reteach**Compare Real Numbers**

Numbers may be classified by identifying to which of the following sets they belong.

Whole Numbers 0, 1, 2, 3, 4, ...

Integers ..., -2, -1, 0, 1, 2, ...

Rational Numbers numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Irrational Numbers numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Examples

Name all sets of numbers to which each real number belongs.

- 1 5 whole number, integer, rational number
- 2 0.666... Decimals that terminate or repeat are rational numbers, since they can be expressed as fractions. $0.666... = \frac{2}{3}$
- 3 $-\sqrt{25}$ Since $-\sqrt{25} = -5$, it is an integer and a rational number.
- 4 $\sqrt{11}$ $\sqrt{11} \approx 3.31662479...$ Since the decimal does not terminate or repeat, it is an irrational number.

To compare real numbers, write each number as a decimal and then compare the decimal values.

Example 5

Replace \bullet with $<$, $>$, or $=$ to make $2\frac{1}{4} \bullet \sqrt{5}$ a true statement.

Write each number as a decimal.

$$2\frac{1}{4} = 2.25$$

$$\sqrt{5} \approx 2.236067...$$

Since 2.25 is greater than 2.236067..., $2\frac{1}{4} > \sqrt{5}$.

Exercises

Name all sets of numbers to which each real number belongs.

1. 30
2. -11
3. $5\frac{4}{7}$
4. $\sqrt{21}$
5. 0
6. $-\sqrt{9}$
7. $\frac{6}{3}$
8. $-\sqrt{101}$

Replace each \bullet with $<$, $>$, or $=$ to make a true statement.

9. $2.7 \bullet \sqrt{7}$
10. $\sqrt{11} \bullet 3\frac{1}{2}$
11. $4\frac{1}{6} \bullet \sqrt{17}$
12. $3.\bar{8} \bullet \sqrt{15}$

LESSON
1.2**Study Guide***For use with the lesson "Apply Order of Operations"***GOAL** Use the order of operations to evaluate expressions.**Vocabulary**

The **order of operations** was established to evaluate an expression involving more than one operation.

Order of Operations**STEP 1** Evaluate expressions inside grouping symbols.**STEP 2** Evaluate powers.**STEP 3** Multiply and divide from left to right.**STEP 4** Add and subtract from left to right.**EXAMPLE 1** Evaluate expressions**Evaluate the expression $4^2 \cdot 5 - 6^2$.****Solution**

$$\begin{aligned}
 4^2 \cdot 5 - 6^2 &= 16 \cdot 5 - 36 && \text{Evaluate powers.} \\
 &= 80 - 36 && \text{Multiply.} \\
 &= 44 && \text{Subtract.}
 \end{aligned}$$

Exercises for Example 1**Evaluate the expression.**

1. $20 - 3^2 + 7$

2. $5 \cdot 2^3 \div 6$

3. $4 \cdot 6 - 21 \div 3$

EXAMPLE 2 Evaluate expressions with grouping symbols**Evaluate the expression.**

a. $47 - 2(9 + 12)$

b. $6[2^3 + (13 - 8)]$

Solution

$$\begin{aligned}
 \text{a. } 47 - 2(9 + 12) &= 47 - 2(21) \\
 &= 47 - 42 \\
 &= 5
 \end{aligned}$$

Add within parentheses.

Multiply.

Subtract.

$$\begin{aligned}
 \text{b. } 6[2^3 + (13 - 8)] &= 6[8 + (13 - 8)] \\
 &= 6[8 + 5] \\
 &= 6[13] \\
 &= 78
 \end{aligned}$$

Evaluate power.

Subtract within the parentheses.

Add within the parentheses.

Multiply.

LESSON
1.2**Study Guide** *continued*
*For use with the lesson "Apply Order of Operations"***Exercises for Example 2****Evaluate the expression.**

4. $3(14 - 5)$

5. $6(9 - 1^4)$

6. $(7 + 5) - (8 + 4)$

7. $(3^3 - 6) \div 3$

8. $4^2(2 + 8)$

9. $9[15 \div (2 + 3)]$

EXAMPLE 3**Evaluate an algebraic expression****Evaluate the expression $\frac{4y + 8}{2 + y}$ when $y = 3$.****Solution**

$$\frac{4y + 8}{2 + y} = \frac{4(3) + 8}{2 + 3} \quad \text{Substitute 3 for } y.$$

$$= \frac{12 + 8}{2 + 3} \quad \text{Multiply.}$$

$$= \frac{20}{5} \quad \text{Add.}$$

$$= 4 \quad \text{Divide.}$$

Exercises for Example 3**Evaluate the expression when $w = 9$.**

10. $17 + 3w$

11. $w^2 - 13$

12. $\frac{5w}{w + 6}$

13. $7(13 - w)$

14. $2w^2 - 15$

15. $5w - \frac{1}{3}w$

LESSON
1.3**Study Guide***For use with the lesson "Write Expressions"***GOAL** Translate verbal phrases into expressions.**Vocabulary**

A **verbal model** describes a situation using words as labels and using math symbols to relate the words.

A **rate** is a fraction that compares two quantities measured in different units.

A **unit rate** is a rate whose fraction has a denominator of 1.

EXAMPLE 1 Translate verbal phrases into expressions**Translate the phrase into an expression.**

- a. 8 more than the product of 5 times a number w
- b. The quotient of 11 and the sum of 7 and a number x
- c. The square of a number y decreased by 13

Solution

Verbal Phrase	Expression
a. 8 more than the product of 5 times a number w	$8 + 5w$
b. The quotient of 11 and the sum of 7 and a number x	$\frac{11}{7 + x}$
c. The square of a number y decreased by 13	$y^2 - 13$

Exercises for Example 1**Translate the phrase into an expression.**

- 1. The difference of 3 times a number m and 5
- 2. 26 divided by a number n
- 3. $\frac{1}{3}$ of a number p
- 4. The sum of 9 and the square of a number k

LESSON
1.3
Study Guide *continued*
For use with the lesson "Write Expressions"
EXAMPLE 2 Use a verbal model to write an expression

A student reads p pages of a 230-page book. Write an expression for the number of unread pages in the book.

Solution

STEP 1 Write a verbal model. Pages in book — Pages read

STEP 2 Translate the verbal model
into an algebraic expression. $230 - p$

An expression that represents the number of unread pages in the book is $230 - p$.

Exercises for Example 2

Write an expression for the situation.

5. Total cost of n notebooks if each notebook costs \$1.25
6. The time it takes to get to school and home again if you walk 5 minutes to the bus stop and ride the bus for m minutes

EXAMPLE 3 Find a unit rate

An airport checks in 460 passengers in 5 hours. Find the unit rate.

Solution

$$\frac{460 \text{ passengers}}{5 \text{ hours}} = \frac{460 \text{ passengers} \div 5}{5 \text{ hours} \div 5} = \frac{92 \text{ passengers}}{1 \text{ hour}}$$

The unit rate is 92 passengers per hour.

Exercises for Example 3

Find the unit rate.

- | | |
|--|---|
| 7. $\frac{129 \text{ miles}}{6 \text{ gallons}}$ | 8. $\frac{18 \text{ people}}{3 \text{ tables}}$ |
| 9. $\frac{\$28}{4 \text{ tickets}}$ | 10. $\frac{1500 \text{ meters}}{7.5 \text{ minutes}}$ |

LESSON
1.4**Study Guide***For use with the lesson "Write Equations and Inequalities"***GOAL** Translate verbal sentences into equations or inequalities.**Vocabulary**

An **open sentence** is a mathematical statement that contains two expressions and a symbol that compares them.

An **equation** is an open sentence that contains the symbol $=$.

An **inequality** is an open sentence that contains one of the symbols $<$, \leq , $>$, or \geq .

When you substitute a number for the variable in an open sentence, the resulting statement is either true or false. If the statement is true, the number is a **solution of the equation**, or a **solution of the inequality**.

EXAMPLE 1 Write equations and inequalities**Write an equation or an inequality.**

- 8 times the quantity of 11 plus a number x is 112.
- The product of 7 and a number y is no more than 31.
- A number z is more than 8 and at most 15.

Solution

Verbal phrase	Equation or inequality
a. 8 times the quantity of 11 plus a number x is 112.	$8(11 + x) = 112$
b. The product of 7 and a number y is no more than 31.	$7y \leq 31$
c. A number z is more than 8 and at most 15.	$8 < z \leq 15$

Exercises for Example 1**Write an equation or an inequality.**

- The difference of 73 and a number x is 17.
- The product of 8 and the quantity of a number y plus 6 is less than 21.
- The quotient of a number w and 5 is at most 4.
- The sum of a number z and 2 is greater than 15 and less than 23.

LESSON
1.4**Study Guide** *continued*
For use with the lesson "Write Equations and Inequalities"**EXAMPLE 2** Check possible solutions**Check whether 5 is a solution of the equation or inequality.**

Equation/inequality	Substitute	Conclusion
a. $3x - 7 = 12$	$3(5) - 7 \stackrel{?}{=} 12$	$8 \neq 12$ ✗ 5 is <i>not</i> a solution.
b. $9 + 2x \leq 23$	$9 + 2(5) \stackrel{?}{\leq} 23$	$19 \leq 23$ ✓ 5 is a solution.

Exercises for Example 2**Check whether the given number is a solution of the equation or inequality.**

5. $13 + a = 17$; 4 6. $7b - 3 = 10$; 2 7. $4c < 15$; 3
 8. $21 - 3d \geq 11$; 2 9. $4g + 6 \leq 14$; 3 10. $7 < m + 8 < 15$; 6

EXAMPLE 3 Solve a multi-step problem

A soccer team is selling pizzas for \$6 each. Each pizza costs \$4 to make. The team has 10 players and wants to raise \$900 for equipment and uniforms. How many pizzas does the team need to sell? How many pizzas will each player sell if every player sells the same number of pizzas?

Solution**STEP 1** Write a verbal model. Let p be the number of pizzas sold. Write an equation.

$$\left(\begin{array}{c} \text{Price of} \\ \text{pizza} \end{array} - \begin{array}{c} \text{Cost to make} \\ \text{each pizza} \end{array} \right) \times \left(\begin{array}{c} \text{Number of} \\ \text{pizzas sold} \end{array} \right) = \text{Profit}$$

$$(6 - 4) \times p = 900$$

STEP 2 Use mental math to solve the equation $(6 - 4)p = 900$, or $2p = 900$. Think: 2 times what number is 900? Because $2(450) = 900$, the solution is 450.

The team needs to sell 450 pizzas.

STEP 3 Find the number of pizzas each player sells: $\frac{450 \text{ pizzas}}{10 \text{ players}} = 45 \text{ pizzas per player}$

Each player will sell 45 pizzas.

Exercise for Example 3

11. Your family is driving 188 miles to visit a relative. Your father drives 63 miles then stops for a break. How many more miles are left in the trip? Your father drives 50 miles per hour. How long will the remainder of the trip take? Write a verbal model for the situation, then solve.

LESSON
1.5**Study Guide***For use with the lesson "Use a Problem Solving Plan"***GOAL** Use a problem solving plan to solve problems.**Vocabulary**A **formula** is an equation that relates two or more quantities.**A Problem-Solving Plan****STEP 1 Read and Understand** Read the problem carefully. Identify what you know and what you want to find out.**STEP 2 Make a Plan** Decide on an approach to solving the problem.**STEP 3 Solve the Problem** Carry out your plan. Try a new approach if the first one isn't successful.**STEP 4 Look Back** Once you obtain an answer, check that it is reasonable.**EXAMPLE 1** Read a problem and make a plan

A group of people go to a play. Adult tickets cost \$8 and tickets for children under twelve years of age cost \$5. There are 4 children under twelve. The group spends \$44 for all the tickets. How many adults attended the play?

Solution**STEP 1 Read and Understand***What do you know?*

You know the cost of each ticket, the number of children attending, and the total cost of the tickets.

What do you want to find out?

You want to find the number of adult tickets purchased.

STEP 2 Make a Plan

Use what you know to write a verbal model that represents what you want to find out. Then write an equation and solve it.

Exercise for Example 1

Identify what you know and what you need to find out. Do *not* solve the problem.

1. A salesman is reimbursed \$50 a day for food and lodging. He also receives \$.35 for each mile driven. He drives 124 miles and is reimbursed \$193.40. How many days was the trip?

LESSON
1.5**Study Guide** *continued*
For use with the lesson "Use a Problem Solving Plan"**EXAMPLE 2** **Solve a problem and look back**

Solve the problem in Example 1 by carrying out the plan. Then check your answer.

Solution

STEP 3 Solve the Problem Write a verbal model. Then write an equation. Let a be the number of adult tickets purchased.

$$\begin{array}{ccccccc} \text{Cost of} & \cdot & \text{Number of} & + & \text{Cost of} & \cdot & \text{Number of} & = & \text{Total} \\ \text{adult tickets} & & \text{adult tickets} & & \text{children's ticket} & & \text{children's ticket} & & \text{cost} \\ 8 & \cdot & a & + & 5 & \cdot & 4 & = & 44 \end{array}$$

The equation is $8a + 20 = 44$. One way to solve the equation is to use the strategy *guess, check and revise*.

Guess a number that seems reasonable considering the total cost of \$44. Try 2.

$$\begin{array}{ll} 8a + 20 = 44 & \text{Write equation.} \\ 8(2) + 20 \stackrel{?}{=} 44 & \text{Substitute 2 for } a. \\ 36 \neq 44 \text{ ✗} & \text{Simplify; 2 does not check.} \end{array}$$

Because $36 < 44$, try a larger number. Try 3.

$$\begin{array}{ll} 8a + 20 = 44 & \text{Write equation.} \\ 8(3) + 20 \stackrel{?}{=} 44 & \text{Substitute 3 for } a. \\ 44 = 44 \text{ ✓} & \text{Simplify.} \end{array}$$

The group bought 3 adult tickets.

STEP 4 Look Back Each adult ticket purchase adds \$8 to the total ticket cost. Make a table.

Number of adults	0	1	2	3	4
Total cost	\$20	\$28	\$36	\$44	\$52

The total cost is \$44 when 3 adult tickets are purchased. The answer in Step 3 is correct.

Exercise for Example 2

Use a problem solving plan to solve the problem.

2. You have saved \$165 to buy a video camera that costs \$300. You plan to save \$15 each week. How many weeks will it take to save for the video camera?

Reteach***The Distributive Property***

- To multiply a sum by a number, multiply each addend by the number outside the parentheses.
- $a(b + c) = ab + ac$
- $(b + c)a = ba + ca$

Example 1 Find 6×38 mentally using the Distributive Property.

$$\begin{aligned} 6 \times 38 &= 6(30 + 8) && \text{Write 38 as } 30 + 8. \\ &= 6(30) + 6(8) && \text{Distributive Property} \\ &= 180 + 48 && \text{Multiply mentally.} \\ &= 228 && \text{Add.} \end{aligned}$$

So, $6 \times 38 = 228$.

Example 2 Use the Distributive Property to rewrite $4(x + 3)$.

$$\begin{aligned} 4(x + 3) &= 4(x) + 4(3) && \text{Distributive Property} \\ &= 4x + 12 && \text{Multiply.} \end{aligned}$$

So, $4(x + 3)$ can be rewritten as $4x + 12$.

Exercises

Find each product mentally. Show the steps you used.

1. 4×82

2. 9×26

3. 12×44

4. 8×5.7

Use the Distributive Property to rewrite each algebraic expression.

5. $5(y + 4)$

6. $(7 + r)3$

7. $12(x + 5)$

8. $(b + 2)9$

9. $4(4 + a)$

10. $9(7 + v)$